

III Semester B.A./B.Sc. Examination, April/May 2021

(Semester Scheme)

(CBCS) (F+R) (2015 – 16 and Onwards)

MATHEMATICS – III

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.**PART – A**

Answer any five questions :

(5×2=10)

1. a) Find all the left cosets of the subgroup $H = \{0, 2, 4\}$ of the group $(\mathbb{Z}_6, +_6)$.

b) Find the number of generators of the cyclic group of order 8.

c) Show that the sequence $\left\{1 - \frac{1}{n}\right\}$ is monotonically increasing sequence.

d) State Cauchy's root test for the series of positive terms.

e) Discuss the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$.f) Discuss the continuity of the function $f(x) = \frac{1}{x^2 - 4}$ at $x = 2$.

g) State Lagrange's mean value theorem.

h) Evaluate $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$.**PART – B**

Answer one full question : (1×15=15)

2. a) If an element 'a' of a group G is of order n, then prove that $a^m = e$ for some positive integer m iff n divides m.b) Define a cyclic group. Show that the multiplicative group of fourth roots of unity is a cyclic group generated by $\langle i \rangle$.

c) State and prove Lagrange's theorem for finite groups.

OR

P.T.O.

3. a) Prove that there is one-to-one correspondence between the set of all distinct right cosets and set of all distinct left cosets of a subgroup of a group.
- b) If G is a cyclic group of order ' d ' and ' a ' is a generator, then prove that a^k ($k < d$) is also a generator of G if and only if $(k, d) = 1$.
- c) Prove that a finite group of prime order is cyclic and hence abelian.

PART - C

Answer two full questions :

(2×15=30)

4. a) Show that the limit of a convergent sequence is unique.

b) Discuss the nature of the sequence $\{x^{\frac{1}{n}}\}$.

c) Test the convergence of the sequences

$$\text{i) } \sqrt{n^2 + 1} - \sqrt{n^2 - 1}$$

$$\text{ii) } \frac{\log(n+1) - \log n}{\sin\left(\frac{1}{n}\right)}$$

OR

5. a) Prove that a monotonic increasing sequence bounded above is convergent.

b) Find the limit of the sequence

$$\{0.3, 0.33, 0.333, \dots\}$$

c) Show that the sequence $\{a_n\}$ defined by $a_1 = 1$, $a_n = \sqrt{2 + a_{n-1}}$ is convergent and converges to 2.

6. a) Discuss the nature of the geometric series $\sum_{n=0}^{\infty} x^n$

b) Discuss the convergence of the series $\frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \dots$

c) Find the sum to infinity of the series $\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \dots$

OR

7. a) State and prove Raabe's test for the series of positive terms.

b) Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$

c) Find the sum to infinity of the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots$

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PART - D

Answer **one full** question :

(1×15=15)

8. a) Show that a function which is continuous in a closed interval attain its bounds.

b) Verify Rolle's theorem for $f(x) = \log\left(\frac{x^2 + 3}{4x}\right)$ in $[1, 3]$. 4

c) Evaluate :

i) $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}}$

ii) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$ 3

OR

9. a) State and prove Cauchy's mean value theorem.

b) Discuss the differentiability of

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases} \quad \text{at } x = 0$$

c) Expand $\log_e(1 + x)$ upto the term containing x^4 using Maclaurin's series.

9
15
16
39